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Abstract

This paper examines the *Antenna Positioning Problem* in cellular networks, involving the selection of a subset of antennas from a larger candidate list, and determining the associated antenna parameters, so as to meet or optimize multiple design criteria. This problem is a critical part of the design process: it constitutes a significant proportion of the network resources involved and it is the first building block of the overall process, hence strongly affecting downstream decisions. The proposed problem formulation attempts to bring together issues raised both in the academic literature and by practitioners, while linking the network infrastructure design phase to the subsequent frequency allocation phase. We propose a tabu search-based algorithm with multiple neighbourhood operators to solve this problem. This heuristic is then evaluated on a real life network. Results show a significant improvement over the currently implemented network configurations in the context of green field and network capacity upgrade scenarios.

Keywords: Telecommunications, GSM networks, antenna positioning, network design, operations research, tabu search method.

Résumé

Cet article traite du problème de positionnement d'antennes dans les réseaux cellulaires, problème qui consiste à sélectionner un ensemble d'antennes parmi une liste d'emplacements possibles et à déterminer leurs paramétrages de façon à optimiser un ensemble de critères. L'importance de ce problème réside dans le fait qu'il est lié à l'utilisation d'une partie importante des ressources du réseau et qu'il intervient en amont du processus de design, affectant ainsi toutes les autres décisions. On propose dans cet article une formulation du problème qui prend en compte des critères de coût, de capacité et de couverture, avec pour objectif de minimiser l'interférence totale. Cette structure du problème nous permet ainsi de faire le lien avec le problème d'affectation de fréquences. Pour la résolution de ce problème, on propose un algorithme de recherche tabou avec de multiples opérateurs de voisinage pour traiter les différents aspects de ce problème. On évalue notre algorithme sur deux jeux de données réelles. Les résultats obtenus améliorent les solutions générées par l'opérateur.

Mots-clés: Télécommunications, réseaux GSM, positionnement d'antennes, conception de réseaux, recherche opérationnelle, méthode de recherche tabou.

1. INTRODUCTION

In this paper we focus on GSM cellular networks as they are the predominant type around the world. A GSM network consists of a set of *antennas* scattered around the service area, each of which covers a small area called a *cell*. A group of *radio transceivers* (TRXs) connected to each antenna send and receive wireless communications to and from *mobile units* using radio frequencies. The *Antenna Positioning Problem* (APP) encompasses a core set of decisions which determine the deployment of physical resources such as antennas and TRXs. Traditionally, the APP is the first step in the network design and management process, the *Frequency Allocation Problem* (FAP) being the next step. The FAP involves the assignment of frequencies from a limited spectrum to the network radios. The quality of the FAP assignments heavily depends on the infrastructure design decisions made upstream while solving the APP. Thus, suboptimal solutions to the APP will translate into largely suboptimal solutions for the overall design process. The rapid growth of communication traffic and the increase in competition make the search for a better utilization of the available resources even more critical.

Currently, the process of locating antennas is mostly manual. It involves the simulation of a few scenarios and makes incremental adjustments until an acceptable solution is found. The development time of a new network configuration, or the upgrade of a network, is often time consuming. A comprehensive model and an automatic positioning and evaluation tool can reduce the duration of the design process, and improve resource utilization.

The APP is defined on a set M of test points which define the work area. The purpose of the APP is to provide a mobile telephone service to these test points. To this end, a candidate antenna set N of potential antenna selections, obtained by varying antenna positions and parameters, is generated and the radio propagation matrix linking N with M is computed. The APP involves selecting a subset from N so as to provide service while managing the system performance criteria such as cost, coverage, capacity, etc.

In the reminder of this paper, we review prior work in Section 2, and present the proposed model and solution approach in Sections 3 and 4. The experimental investigation is reported in Section 5.

2. LITERATURE REVIEW

A few models (e.g., Reininger et al., 1999) consider a tactical planning approach to provide a time plan for the development of the network. Most available models examine the APP from an operational standpoint where the aim is to design a network for a particular point in time. This is the approach taken in this paper. The emphasis on operational models may be explained by the uncertainty associated with generating the problem data over a long planning horizon. We refer the reader to Touhami (2004) for a more comprehensive review of the literature on APP models. These models differ mostly in their definition of the elements of the problem: the work area, the candidate antenna set, and the system performance criteria.

The work area is defined by a spatial partitioning of the area for which the operator wishes to provide wireless telephone service, producing the set N of test points. Different types of partitioning rules have been proposed. For instance, Tutschku (1999) and Mathar and Niessen (2000), use a partition based on the demand node concept. "A demand node represents the center of an area that contains a quantum of demand from teletraffic viewpoint, accounted in a fixed number of call requests per unit time" (Tutschku, 1998a). Other models use a grid where the test points correspond to areas of equal measure. The model proposed in this paper can work with both types of partitioning but we recommend the second approach: when using a demand based partitioning, a demand node with low traffic density will tend to cover a larger area. Variations in signal strength in such extended areas may be significant, and hence the radio propagation matrix may not accurately represent the real network.

Demand distribution is another aspect of the work area that may be incorporated in the APP. Most models consider a non-uniform traffic distribution either by using the demand node concept (e.g., Mathar and Niessen, 2000) or by assigning a weight to each test point (e.g., Bose 2001, Mazzini et al. 2003, and Zimmermann et al. 2003). Models that do not consider variations of traffic across the service area often aim at maximizing

the size of the area served or at minimizing costs (e.g., Krishnamachari and Wicker 20000, Whitaker et al. 2004). In this case it is assumed that the problem of determining the capacity of the equipment is to be solved at a later stage. While lacking accuracy, such models are more suited for the earlier design phases.

Each potential antenna in the network can be described by its parameters which may include antenna type, coordinates, height, azimuth, tilt, transmission power, maximum capacity, etc. Thus, the candidate antenna set is a set of combinations of values for these parameters. The size of the set is controlled by limiting the number of parameters under consideration, and setting the remaining parameters at some average level or by reducing the range of values that the parameters can take. This approach results in reducing the candidate antenna set size but at the expense of overlooking some potentially interesting configurations during the optimization process. Another approach would be to include the selection of the value of any of the antenna characteristics as an additional decision variable. Therefore, this approach adds to the complexity of the problem which could be overcome in part by designing an appropriate optimization algorithm. In the model described in this paper, we are given the candidate antenna set and, for each antenna, a set of power levels at which it can operate. The model has then to determine the transmission power level so as to optimize the performance of the system. Because this parameter is critical, it cannot be simply set at some average value nor can all possible power levels be considered because of the significant increase in the size of the candidate antenna set.

The radio propagation or pathloss matrix describes the links between the candidate antenna set N and the set of test points M. Because the signal is transmitted from a TRX attached to an antenna, its strength decreases as a function of distance, terrain characteristics, and clutter in the area. The *antenna print* is the set of points receiving a signal from the antenna. The *cell* associated with each antenna is the part of the print in which it generates the strongest received signal. By varying the antenna parameters within the allowed range, the print of the antenna will also change. Figure 1 shows the print of two antennas which only differ in azimuth. This figure shows a three-dimensional view of the intensity of the received signal. Note the non-uniformity of wave propagation in all dimensions and the significant impact of the change in azimuth. The

area with received signal less than a given threshold (set at -92dBm for the networks tested in this paper) is not considered covered by the antenna as this signal is considered too weak to carry calls. This example shows how the effect of the transmission of one antenna may reach a larger area far beyond its cell and thus potentially cause significant interference. In fact a signal below -92dBm, though weak, can still cause interference, especially when combined with the other non-serving signals. It is also worth mentioning that the change in signal strength in the downlink direction (from the antenna to the mobile user) and the uplink direction is not symmetrical. Most articles consider only the downlink direction and assume that either both directions are symmetrical or that care has been taken within the pathloss matrix to compensate for asymmetry (e.g., Reininger and Caminada, 1998).

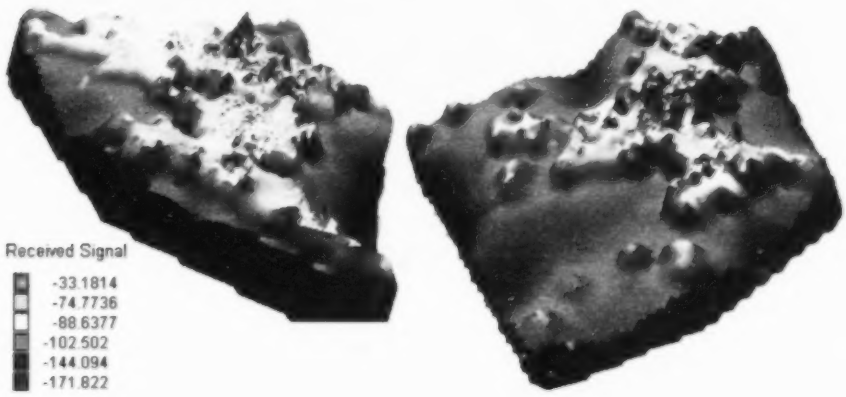

Figure 1: Sample cell prints

The final building block of APP models is the system performance measures which involve costs and quality of service. The latter can be further decomposed into system coverage, capacity and interference. The system costs are estimated in terms of the number or value of sites, antennas, or TRXs. Models that incorporate cost considerations do so either by imposing a constraint on the number of resources that can be deployed (budget constraint) (e.g., Han et al. 2001, Mathar and Niessen 2000, and

Mathar and Schmeink 2000) or by minimizing the use of these resources within the objectives function (e.g., Mazzini et al. 2003, Whitaker et al. 2004).

The network *coverage* refers to the area or number of subscribers that can receive signals strong enough to carry communications. Most models use this criterion as the objective function, although they sometimes require a coverage exceeding a specified threshold as a constraint, often set at 100% coverage (e.g., Hurley 2000,Mazzini et al. 2003. Molina et al. 2000. and zimmermann et al. 2003). In this paper, we consider the latter approach.

Connectivity is another feature related to network coverage: as a mobile user moves from the coverage area of one antenna to another, the handoff process allows the communication to remain active. To achieve this, the cells need to be connected (e.g., Reininger and Caminada (1998)). Vasquez and Hao (2001) observe that the handoff constraint is relatively easy to satisfy provided there exists a sufficient number of cells in the network to provide coverage.

Once a mobile user is covered by the network, the next question is whether enough TRXs are installed in the cell to carry the communication into the network. Most models do not consider this issue of *capacity*, as it is viewed as a separate problem that follows the resolution of the APP. In this paper, we incorporate capacity planning within the APP using the same approach as Vasquez and Hao (2001). The main reason is to avoid cases where the APP solution generates cells covering a large volume of traffic in excess of the maximum antenna capacity. Our experiments indicate that the optimization is sensitive to capacity-related constraints, making the case for managing capacity planning within the APP. Since this involves a simple procedure (see Section 3 for details), it is not likely to increase in problem difficulty.

The last criterion for the quality of service is related to the level of *interference* which occurs when two communications are established on the same frequency or on adjacent frequencies, resulting in co-channel or adjacent channel interference, respectively. Solving the FAP will result in an assignment of frequencies from the available spectrum so as to minimize interference (Touhami, 2004). Therefore, the effective interference can be measured only after an FAP solution is obtained. Only a few models propose to simultaneously solve the APP and the FAP (Floriani and Mateus

1997a, Mathar and Niessen 2000, Mazzini et al. 2003, and Molina et al. 2000). Due to the significant increase of problem size and difficulty, the standard reuse schemes for frequency allocation are employed. These are based on pattern repetition and do not perform nearly as well as free optimization of the FAP. Most articles fall within the sequential decomposition framework where the APP is solved under the assumption that all antennas are operating on the same frequency. The FAP is solved in a later phase. The resulting network is suboptimal but the solution process is more tractable and allows for the handling of larger networks. In this context, when interference is considered in the APP, two main approaches are used. In the first, interference handled implicitly by minimizing the cell overlap area (e.g., Hurley 2000, and Tutschku et al. 1997), by minimizing the sum of interfering signals (e.g., Zimmermann et al., 2003), or by maximizing the strength of the signal carrying the communication (e.g. Sherali et al., 1996). In the second, interference is handled explicitly by minimizing its total value (e.g., Vasquez and Hao, 2001), or by imposing a constraint on its maximum allowable value (e.g., Tutschku et al. 1999, and Whitaker et al. 2004). The importance of interference management is highlighted by the large body of the FAP models that consider it as the main objective. The size of today's networks is such that interference cannot be avoided but rather needs to be managed. For this, the model presented in this paper explicitly incorporates interference measurements as the objective function.

In most models that explicitly consider interference, the *carrier to interferer ratio* (CIR) is used as the measure of interference. At any point in the service area at which multiple signals are received, the CIR is defined as the ratio of the signal carrying the communication to the sum of all other signals acting as interferers. This measure of interference is the easiest to compute and integrate in APP models. Other measures, such as the bit error rate, are more difficult to compute.

It is worth noting at this stage that the emphasis of several articles reviewed is more on modeling than on problem solving. This is perhaps indicative of the fact that a generally comprehensive and accepted model structure has not yet been developed. Although most articles comment on the complexity of the proposed models, few complexity proofs are presented. Glaßer et al. (2000) studied two APP models for minimizing the number of antennas subject to coverage constraints, and proved the NP-

completeness of these two problems. Most articles use heuristics to overcome the combinatorial and non-linear nature of the APP. A wide range of heuristic optimization techniques have been used, such as tabu search (Vasquez and Hao 2001, Krishnamachari and Wicker 2000), simulated annealing (Hurley 2000, and Mathar and Niessen 2000), genetic algorithms (Han et al. 2001, and Whitaker et al. 2004), evolutionary algorithms (Raisanen and Whitaker 2003, and Zimmermann et al. 2003), greedy algorithms (Tutschku 1998a and Vasudevan and Rasmussen 1999), and fuzzy expert systems (Huang et al. 2000). Exact solution approaches are employed in some cases such (e.g., Floriani and Mateus 1997b, and Mazzini et al. 2003) but are validated on relatively small instances. It is difficult to fully assess the performance of the proposed algorithms because of the small size of the networks tested and especially the lack of benchmark instances.

3. MODEL

We now present a problem formulation in which we attempt to integrate a number of design concepts of concern to network operators: coverage, capacity, interference and costs. Before presenting the model, we start by describing the constituting elements of the problem: the working area, the antenna set, and design criteria.

3.1 Work area

The model requires a partitioning of the service area into a grid of test points. The impact of any network configuration would then be evaluated at every point. These evaluations are aggregated in order to obtain an overall evaluation of the proposed configuration. For each point on the grid, we define:

N : the set of test points of interest;

a_n : the volume of traffic offered at test point n;

p_{in} : the strength of the signal transmitted by antenna i and received at test point n, expressed in dBm;

q_{in} : the degradation in signal strength from antenna i received at test point n, expressed in dBm;

p_{min}: the threshold on acceptable signal strength, expressed in dBm; for a communication to take place at a particular test point, we need $p_{in} > p_{min}$;

$y_{in} = 1$, if $p_{in} = \max\{p_{jn}\}, p_{in} > p_{min}, 0$, otherwise; this variable indicates if antenna i is the server of test point n by providing the strongest signal;

γ_n: the interference level at test point n, expressed in terms of CIR;

γ^l: interference threshold, below which CIR is considered too low;

$z_n = 1$, if $\gamma_n \leq \gamma^l, 0$, otherwise. This variable indicates if test point n is subject to high interference.

3.2 Candidate Antenna Set

The decision variables of the model define the selection of a subset of antennas from the candidate antenna and their transmission power. The candidate set includes a subset of the possible combination of antenna characteristics. We do not impose any restriction on this set. The cell print that results from an antenna in the candidate list is computed off-line and presented as an input to the model in the form of the path loss matrix. The proposed model also associates a cost term with every activated TRX, antenna, and site which can accommodate multiple antennas.

Table 1: Capacity and number of TRXs at 2% blocking, 0.022 Erlang per user

Number of TRXs	1	2	3	4	5	6	7
Traffic carried (Erlang)	2.93	9.00	15.76	22.82	30.05	37.45	44.90
Number of subscribers served	132.1	405.5	709.3	1027.3	1352.4	1685.4	2020.7

Every solution to the APP provides an assignment of test points to the antennas providing the strongest signal. Thus for each APP solution, one can compute the total traffic volume associated with each antenna. As a consequence, each antenna can be viewed as a queueing system where users are jobs to be processed and TRXs are system servers. Hence, determining the number of TRXs per antenna (i.e., capacity planning) is equivalent to computing the number of servers needed. This system is often modeled as

an M/M/s queue with infinite sources and where blocked requests are lost. For technical reasons, this system also has a limit on the maximum number of TRXs/servers that can be installed. For the example used in this work, a maximum of 4 or 7 TRXs can be associated with every antenna. Table 1 shows the traffic volume that can be served as a function of the number of TRXs installed for a 2% blocking probability, where traffic is expressed Erlang or in the number of subscribers (0.022 Erlang per subscriber). If the offered traffic exceeds the capacity of provided by the TRXs installed, then the excess traffic is not carried and is blocked.

Using conversion Table 1, and assuming a maximum of 4 TRXs per antenna, we define the following:

M : the set of antennas in the candidate list;

K : the set of sites, corresponding to the antenna set in the candidate list;

$x_i = 1$ if antenna i is active, 0 otherwise: these decision variables indicate if an antenna is active;

p_i: the transmission power of antenna i, expressed in dBm. Thus, for any test point n, the received signal from antenna i is $p_{in} = p_i + q_{in}$;

Δ_i: the discrete set of possible values of p_i for antenna i. For instance, for some antenna i we may have $\Delta_i = \{off, 45, 46, 47, 48\}$ (dBm);

$w_k = 1$ if $x_i = 1$ for some $i \in k$, 0 otherwise: this is the indicator variable associated with every site;

$o_i = \sum_{n \in N} a_n y_{in}$: the offered traffic at antenna i for a given configuration;

$d_i = 1$ if $o_i \le 132.1$, 2 if $132.1 < o_i \le 405.5$, 3 if $405.5 < o_i \le 709.3$, 4, otherwise: the number of TRXs to be installed at antenna i;

$b_i = 0$ if $o_i \le 1027.3$, $o_i - 1027.3$ otherwise: the blocked traffic at antenna i. This corresponds to the traffic offered to antenna i but that cannot be carried because not enough TRXs could be installed;

$c_i = \{o_i$ if $o_i \le 1027.3$, 1027.3 otherwise: the carried traffic at antenna i. This corresponds to the traffic served by antenna i that could be processed by the installed TRXs;

$cap_i = 132.1$ if $d_i = 1, 405.5$ if $d_i = 2, 709.3$ if $d_i = 3, 1027.3$ if $d_i = 4$: the capacity of antenna i, expressed as the total number of subscribers that can be served by the TRXs installed;

$e_i = 132.1 - o_i$ if $b_i = 1, 405.5 - o_i$ if $b_i = 2$, $709.3 - o_i$ if $b_i = 3$, $1027.3 - o_i$ if $b_i = 4$, 0 otherwise: the excess capacity at antenna i. This corresponds to additional traffic that the currently installed number of TRXs could carry;

c_i^a: the cost of antenna i;

c_k^s: the cost of site k;

c^t: the cost of a single TRX t.

3.3 Network design criteria

A network configuration S is defined by the set of active antennas and their selected transmission power. Thus, S is defined as

$$S = (X, P) = \{(s_i, p_i),\ i \in M\}. \quad (1)$$

The evaluation of any network configuration, S, is based on four dimensions: the network coverage, the network capacity, the network interference, and the network cost and profit. Any of these criteria can be incorporated in the objective function or in the problem constraint set.

3.3.1. Network coverage and capacity

We use the following definitions to describe capacity and coverage-related performances of any given configuration S:

$A(S) = \sum_{i \in M} x_i$: the total number of active antennas in configuration S;

A^*: the maximum number of antennas that can be activated;

$U(S) = \sum_k w_k$: the total number of active sites in configuration S;

U^*: the maximum number of sites that can be opened;

$T(S) = \sum_{i \in M} d_i$: the total number of installed TRXs in configuration S;

T^*: the maximum number of TRXs that can be installed;

$Cov(S) = \sum_{i \in M} \sum_{n \in N} y_{in}$: the total covered area in configuration S;

Cov_*: the minimum required level of coverage, expressed as a percentage of the total area;

$O(S) = \sum_{i \in M} o_i$: the total network offered traffic in configuration S. This term measures the traffic coverage of configuration S and provides an estimate of the maximum revenue that can be generated by S;

O_*: the minimum required covered traffic volume, expressed as a percentage of the total offered traffic;

$C(S) = \sum_{i \in M} c_i$: the total network carried traffic in configuration S. This term corresponds to the fraction of offered traffic that can be served by the installed TRXs;

C_*: the minimum required carried traffic, expressed as a percentage of the total offered traffic;

$B(S) = \sum_{i \in M} b_i$: the total network blocked traffic in configuration S. Traffic is blocked when not enough TRXs could be installed at some antenna. This terms measures lost revenues as well as quality of service. Blocked communications may be frustrating to customers, and hence affect the perceived quality of service;

B^*: the maximum allowed blocking, expressed as a percentage of the traffic covered;

$Cap(S) = \sum_{i \in M} cap_i$: the total network capacity in configuration S. This term describes the total number of customer that can be served by the proposed configuration. S;

$E(S) = \sum_{i \in M} e_i$: the total network excess capacity in configuration S. This term describes the equipment efficiency of the proposed configuration and provides an estimate of the slack resources built in the proposed configuration (which could be seen as provisions for future network development);

E^*: the maximum allowed excess capacity, expressed as a percentage of total network capacity.

3.3.2. Network interference

Assuming all signals are on the same frequency (co-channel interference case), the CIR level at any test point n is defined as the ratio of the serving signal to the combined effect of all interfering signals on the same frequency. Since all power variables are expressed in dBm, the CIR function needs to be adjusted. The conversion from mW into dBm is given as follows: $P(dBm) = 10\log_{10}[P(mW)/P_0(mW)]$, where P_0 is the reference power (one mW). Hence, the CIR at any test point n is given by:

$$\gamma_n = \sum_{i \in M} p_{in} y_{in} - 10\log_{10} \sum_{j \in M} 10^{p_{jn}(1-y_{jn})/10} \tag{2}$$

where

$$p_{in} = x_i(p_i + q_{in}) \tag{3}$$

$$\sum_{i \in M} y_{in} \le 1. \tag{4}$$

The first term in equation (2) identifies the server signal of the test point under consideration, while the second term computes the combined effect of all interfering signals. Note that since power terms are expressed in dBm, they need to be transformed back to mW and added to determine their combined effect. Equation (3) is used to compute the received signals at the test points, given the signal transmitted at the antennas. Equation (4) ensures that a test point has no more than one serving antenna. The CIR at all test points should be as high as possible. Thus, for a given configuration, two possible interference-related design criteria would be: the average CIR and the average weighed CIR (weighed by the offered traffic), which one would like to maximize. Hence, we define:

$Q(S) = \sum_{n \in N} \gamma_n / Cov(S)$: the average CIR across all covered test points in N, and

$TQ(S) = \sum_{n \in N} a\gamma_n / O(S)$: the average weighed CIR across covered traffic in N.

Operators only have a limited radio spectrum at their disposal. For realistic size networks, increasing CIR would come at the expense of deploying more resources or reducing covered or carried traffic. In addition, at some CIR levels the communication quality is sufficiently acceptable and the benefits of a further CIR increase are imperceptible. The downside of maximizing average CIR is that the solution may have test points with unnecessarily high CIR levels while others suffer from significant interference levels. Therefore, another possible interference-related design criterion would be to minimize the area (or traffic) for which the CIR is too low. In fact, this is the same type of criterion used in the FAP. After consulting with network design engineers, the threshold value, γ^l, was set at $\gamma^l = 12$ dBm. Hence, we define:

$R(S) = \sum_{n \in N} z_n$: the number of test points with CIR$< \gamma^l$, and

$TR(S) = \sum_{n \in N} a_n z_n$: the total traffic with CIR$< \gamma^l$.

3.3.3. Network costs

Operating costs and profit constitute the fourth dimension of evaluation for a network configuration. We define:

$Cost(S) = \sum_{k \in K} c_k^s w_k + \sum_{n \in N} c_i^a x_i + \sum_{m \in M} c^t d_i$: the total network operating costs.

This corresponds to the summation of site, antenna and TRXs costs, and

$Cost^*$: the total available budget.

Above we have listed a variety of design criteria that need to be managed simultaneously in order to obtain good APP solutions. For each criterion, we associate a target performance level. We can define Ω as the set of solutions achieving those target performance levels:

$$\Omega = \{S : \text{such that} : Cov(s) \geq Cov_* \cdot |N|, O(S) \geq O_* \cdot \sum_{n \in N} a_n, C(S) \geq C_* \cdot \sum_{n \in N} a_n,$$
$$B(S) \leq B^* O(S), E(S) \leq E^* Cap(S), Cost(S) \leq Cost^*, \quad (5)$$
$$A(S) \leq A^*, U(S) \leq U^*, T(S) \leq T^*\}$$

Note that the above definition of Ω does not include any interference-related design criterion. We have opted to include such criteria in our model within the objective

function. This is because by trying to minimize interference within the objective function, we link the APP to the FAP which uses a similar interference reduction objective. Including interference considerations as constraint requires the determination an appropriate value for the minimum average CIR or the maximum area with low a CIR. This is not easy to achieve. Such considerations are more a consequence of the available resources and of the business goals of the network operator. Any of the combination of the above mentioned criteria can be used as objective function but because of these arguments, we define the objective function as $F(S) = TR(S)$.

The next section describes the proposed model for the APP under the assumption that all antennas use the same frequency, and so we refer to this it as the *single frequency model* (SF-APP).

3.4 Single Frequency APP Formulation

In this formulation, we model the APP under the assumption that all antennas operate with the same frequency. The proposed formulation is as follows:

(SF-APP) Optimize $F(S)$ (6)

subject to

$p_{in} = x_i(p_i + q_{in}) \quad \forall\, i \in M \text{ and } n \in N$ (7)

$0 \le (p_{in} - p_{\min}) y_{in} \quad \forall\, i \in M \text{ and } n \in N$ (8)

$p_{in} \ge p_{jn} - L(1 - x_i) - L(1 - y_{in}) \quad \forall\, i, j \in M \text{ and } n \in N$ (9)

$1 \ge \sum_{i \in M} y_{in} \quad \forall n \in N$ (10)

$\gamma_n = \sum_{i \in M} p_{in} y_{in} - 10 \log_{10} \sum_{j \in M} 10^{p_{jn}(1 - y_{jn})/10} \forall n \in N$ (11)

$Lz_n \ge (\gamma^1 - \gamma_n) \quad \forall n \in N$ (12)

$w_k \ge x_i \quad \forall i \in k \text{ and } \forall k \in K$ (13)

$S \in \Omega$ (14)

$p_i \in \Delta_i \quad \forall i \in M$ (15)

$x_i, y_{in}, z_n, w_k \quad \forall n \in N,\ \forall\, i \in M, \text{and } \forall k \in K$. (16)

Equation (7) is used to compute received signals at every test point from all active antennas. A test point is considered covered by an antenna if the received signal is strong enough (inequalities (8)) and the signal is the strongest signal among all other received signals (inequalities (9), where L is a sufficiently large constant). Constraints (10) indicate that a test point has no more than one server. The CIR level at any test point is calculated according to equation (11). Inequalities (12) are used to identify test points with CIR below γ^1. Inequalities (13) are used to determine the set of active sites corresponding to the set of active antennas. Condition (14) represents the constraint set induced by the desired network performance.

4. PROPOSED HEURISTIC ALGORITHM

The description of the APP provided in the previous section highlights the complexity of the problem. Heuristic optimization provides the flexibility needed to manage the variety of design criteria while handling non-linearity. We propose to use a tabu search algorithm in which the search operators are adapted to the problem at hand.

4.1 Solution Evaluation

A network configuration $S = (X, P) = \{(s_i, p_i) \forall i \in M\}$ is determined by the set of active antennas, their corresponding selected transmission power. Denote by $Perf_c(S)$ the performance of configuration S with respect to a performance criterion $c \in \Omega$. We also denote by T_c, the target performance set by the operator. For instance, c may be the cost associated with configuration S and T_c may the total available budget. The constraints that define Ω are of the form $Perf_c(S) \leq T_c$, indicating that the performance of solution S with respect to design criterion c should not exceed a target level T_c. For each such constraint we associate a weight λ_c. For each solution we have already associated an objective function $F(S)$. We also define:

$$\Pi(S) = \sum_c \lambda_c \max\{0, (Perf_c(S) - T_c)/T_c\}$$

which describes the degree of feasibility of the solution S, with respect to all the constraints that define Ω. Note that for all feasible solutions, we have $\Pi(S) = 0$. We also define the compounded objective functions:

$$G(S) = F(S) + \Pi(S),$$

where $G(S)$ is used to compare solutions within the optimization process. By dynamically varying the weights λ_c during the optimization, one can direct the algorithm to seek feasibility with respect to the most violated constraints. This also can serve as a diversification mechanism.

The complete evaluation procedure of a solution S can be summarized as follows:

Procedure Evaluate (S)

1. For all test points n: determine best servers: i_n, and compute γ_n.
2. For all active antennas j: compile all variables ($o_j, d_j, b_j, c_j, cap_j, e_j$).
3. Compute $F(S)$ and $G(S)$.

The evaluation procedure requires a number of loops across test points and antenna set and is therefore relatively time consuming. Thus, an extensive evaluation of the search space would be costly. The search algorithm proposed in this paper attempts to direct the search so as to reduce the number of solution evaluations.

4.2 Solution neighbourhood

The search space of SF-APP as defined in the previous sections, consists of all possible assignments of x_i and p_i. Therefore, the size of the search space is $\Pi_{i \in M} |\Delta_i|$. Our algorithm explores the (x_i, p_i) search space through the application of a sequence of moves. An elementary modification to a solution would be the increase or decrease of the power level of a transmitter in the candidate list. Recall that $\Delta_i = \{p_i^0, .., p_i^{|\Delta_i|}\}$ is defined as the discrete set of feasible power levels of an antenna i, where $p_i^0 < p_i^1 < .. < p_i^{|\Delta_i|}$ and

p_i^0 corresponds to the antenna being off. We consider as an elementary modification to a solution S the increase or decrease of the power level of an antenna i from it current level p_i^k to the next feasible level. We denote an increase in power by $move(i, p_i^{k+1})$ and a decrease as $move(i, p_i^{k-1})$. We use the symbol $\oplus$ to denote the application of an elementary modification to a solution S. Thus a solution $S' := S \oplus move(i, p_i^{k+1})$ is identical to S, except for transmitter i for which the power level is increased to the next feasible level.

Because the CPU time required to evaluate a solution is very high, examining all such solutions in the neighbourhood of the current solution is costly. Thus, we have opted to use a reduced neighbourhood. For this, the proposed algorithm exploits the anticipated local effects of manipulating antenna power. Suppose we are given a configuration S. Keeping everything else unchanged, increasing the power level of an antenna i may result in:

- increasing the area and traffic covered by i, increasing number of TRXs required, or increasing traffic blocking (and the opposite effect may occur in an adjacent antenna);
- improving the received signal of the covered test points, improving their CIR;
- increasing received signal strength of test points belonging to some adjacent antennas, deteriorating their CIR;
- opening a new site, or increasing the number of active antennas if i was inactive.

The reverse local effects may take place if the antenna power is reduced. The proposed algorithm reduces the number of solutions evaluations by exploiting these expected effects to direct the navigation in the search space so as to favour some particular characteristic of the solution or restore feasibility. The combined effect of directed selection of the solution neighbourhood and the penalty terms of the objective function can be expected to improve the algorithm's capacity to identify interesting solutions and, as a consequence, improve its speed. Experiments with neighbourhood reduction techniques based on neighbourhood sampling did not perform as well.

The proposed algorithm alternates between two search phases: the constraint management phase and the system stabilization phase. In the first phase, the aim is to decrease the infeasibility of the current solution with respect to some of the violated constraints in Ω. In the system stabilization phase, the focus is on optimizing the objective function.

4.2.1. The constraint management phase

Infeasibilities with respect to these constraints in Ω are managed by the procedures listed below. In the following, the intersection between a pair of antennas refers to the set of test points that receive a signal from both antennas.

- ***Procedure improve network area coverage***

To increase network coverage, one needs to increase the coverage area of some antenna, by increasing power.

1. Select a random portion A of the total area.
2. Increment a counter function for each antenna that can serve an uncovered test point in A.
3. With a probability proportional to the counter function, select a maximum of 3 antennas i whose status is not tabu, assign $N(S) := N(S) \cup S \oplus move(i, p_i^{k+4})$.

- ***Procedure improve network traffic coverage***

This procedure is similar to improving the area coverage except that the counter function is incremented by the volume of test point traffic.

- ***Procedure reduce blocked traffic***

When traffic is blocked then some antenna needs to decrease its power so as to reduce the antenna coverage and increase the power of some of adjacent antennas so as to cover some of the newly uncovered traffic.

1. Select some antenna i with blocked traffic (with a probability of selection proportional to blocked traffic), set $N(S) := N(S) \cup S \oplus move(i, p_i^{k-4})$
2. Identify all antennas j that can potentially intersect with i.

3. With a probability proportional to the intersection area, select a maximum of 3 antennas j and assign $N(S) := N(S) \cup S \oplus move(j, p_j^{k+4})$ when not tabu.

- ***Procedure Reduce excess capacity***

Excess capacity can be reduced by increasing the offered traffic at the antenna and by decreasing the power of some adjacent antenna.

1. With a probability proportional to excess capacity, select antenna i: $N(S) := N(S) \cup S \oplus move(i, p_i^{k+4})$.
2. Identify all antennas j that can potentially intersect with i.
3. With a probability proportional to the intersection area, select the 3 antennas j whose status is not tabu, assign $N(S) := N(S) \cup S \oplus move(j, p_j^{k-4})$.

- ***Procedure reduce excess number of sites***

This procedure identifies a site that contains only one active antenna and adds a move to disactivate it.

1. From the set of all sites with only one active antenna, probabilistically select one site with active antenna i whose status is not tabu.
2. Add a move to disactivate antenna i: $N(S) := N(S) \cup S \oplus move(i, p_i^0)$.

- ***Procedure reduce excess number of antennas***

The excess number of antennas can be reduced by disactivating an antenna. With a probability of selection inversely proportional to the volume of offered traffic, disactivate an antenna i: $N(S) := N(S) \cup S \oplus move(i, p_i^0)$.

Reducing cost can be achieved by calling the procedures to reduce the number of sites or the number of active antennas, or by reducing the excess capacity. Increasing the volume of carried traffic is achieved by calling the procedures to reduce blocked traffic or improving traffic coverage.

The constraint management phase combines the above described procedures to regain solution feasibility. This can be summarized as follows:

Constraint management phase

1. For the specified number of iterations
    a. With a probability of selection proportional to the degree of infeasibility, select an infeasible constraint in Ω (among all constraints not satisfied by all solutions visited during the previous system stabilization phase).
    b. Using the appropriate procedures described above, build $N(S)$.
    c. Implement all moves in $N(S)$.

Note that at each iteration of the constraints management phase, all identified moves are implemented. This is because experimental analysis indicated that in order for these moves to produce the desired effect (reduce infeasibility), the combined action of the multiple moves is often needed. In addition, only constraints that have not been satisfied by all solutions generated during the previous system stabilization phase are considered. This means that the constraint management phase only intervenes when the penalty terms in the objective function fail to restore feasibility.

4.2.2. The system stabilization phase

During the system stabilization phase, the focus is on improving the system objective function. This phase employs three procedures to generate the moves so as to manage the interference-based objective function.

- ***Procedure for interference management 1:***

This procedure identifies a list of antennas for which to increase power and another list for which to decrease power.

1. Associate a weight φ_i with each candidate antenna i and initialize $\varphi_i = 0$.
2. For all test points n with current server i^* and $\gamma_n < \gamma^1$:
    a. $\varphi_{i^*} := \varphi_{i^*} + 1$ (favour a power increase of server).
    b. For all antennas $j \neq i^*$ (with transmission power at p_j^k).

i. If $p_j^k + q_{jn} > p_{i^*}^k + q_{i^*n}$ (case increasing power of j, makes it a better server of test point n), then $\varphi_j := \varphi_j + 1$.

ii. Else, $\varphi_j := \varphi_j - 1$ (favour a power increase of j).

3. With probability proportional to φ_i, select two non-tabu antennas: $N(S) := N(S) \cup S \oplus move(i, p_i^{k+1})$.
4. With probability inversely proportional to φ_i, select two non-tabu antennas: $N(S) := N(S) \cup S \oplus move(i, p_i^{k-1})$.

- ***Procedure for interference management 2:***

This procedure measures pairwise interference from adjacent active antennas and reduces the power of those causing most of the interference.

1. Randomly select an active antenna i and identify the set of active antennas j causing interference on i.
2. Associate a weight $\varphi_i(j)$ with each candidate antenna j, and initialize $\varphi_i(j) := 0$.
3. For all identified antennas j, and for all test points n with current server i: compute $\varphi_i(j) := \sum a_n(p_{in} - p_{jn})$.
4. For antenna i, $N(S) := N(S) \cup S \oplus move(i, p_i^{k+1})$.
5. With probability inversely proportional to $\varphi_i(j)$, select 3 non-tabu antennas j: $N(S) := N(S) \cup S \oplus move(i, p_i^{k-1})$.

- ***Procedure: interference management 3:***

Another approach to reduce the interference levels is to reduce the coverage of some existing antennas and add a new one. Suppose that a set of test points covered by some antenna i with low CIR due to an antenna j. The idea here to activate a new antenna h and reduce the power levels of antennas i and j.

1. Randomly select an active antenna i and an active antenna j causing interference on i.
2. With probability proportional to the intersection area, select antenna h that intersects both i and j.
3. Set: $N(S) := N(S) \cup S \oplus move(i, p_i^{k-1})$, $N(S) := N(S) \cup S \oplus move(j, p_j^{k-1})$ and $N(S) := N(S) \cup S \oplus move(h, p_h^{k+1})$, when not tabu.

The system stabilization phase uses the interference management procedures described above to identify a list of solution neighbours that may potentially lead to improving solutions. In addition, in order to avoid a significant departure from feasibility, this phase occasionally calls the procedures used within the constraint management phase. The set of solutions in the neighbourhood solutions are evaluated in random order, with the exception of moves which turn off active antennas which are placed first in the list. This introduces a bias towards using fewer resources.

Procedure: System stabilization phase

1. For the specified number of iterations
    a. With uniform probability select an interference management procedure.
    b. Select an infeasible constraint in Ω with a probability proportional to the degree of infeasibility and select an appropriate procedure.
    c. Shuffle the list of moves generated.
    d. Implement all improving moves among all proposed moves.

4.3 Tabu list management and parameter updating

The variations embedded in the move generation procedures and the number of operator in the algorithm make cycling between solutions unlikely. A tabu list is also incorporated in the proposed algorithm in order to avoid the remaining rare cases of cycling that may still occur. For each antenna in the candidate set we associate a tabu tag indicating the iteration number until which all modifications to the antenna are forbidden. The antenna status is declared tabu for the next 20 iterations after a change in its power

level has been made. The variations embedded in the move generation procedures allow us to use short tabu durations. Initial experimental runs indicated that long tabu durations tend to lead the algorithm into exploring a large number of solutions that are not interesting.

The weights λ_c associated with each constraint incorporated in $\Pi(S)$ are updated dynamically during the progress of the algorithm so as to restore solution feasibility. This works in conjunction with the variable move generation procedures described above. The updating takes place after each system stabilization phase, for each constraint c used to define Ω: If all solutions visited during the previous system stabilization phase do not satisfy constraint c, then $\lambda_c := 2\lambda_c$, else, $\lambda_c := 0.5\lambda_c$.

4.4 Intensification and diversification procedures

The intensification procedure is called when a move on antenna i has produced a new best solution. The subset of antennas, intersecting with i, is examined for possible gains by evaluating a possible increase or decrease in power level and implementing all improving changes.

A first diversification mechanism is triggered by a failure to improve upon the best feasible solution. The procedure involves a random modification to the current solution by activating a new site and disactivating an existing site. A second type diversification takes places after failure to visit feasible solutions for a fixed number of iterations. This procedure attempts to exploit any slack resources to improve solution feasibility by activating as many new antennas as possible as long as this improves the degree of feasibility of the current solution. In addition, in order to redirect the search towards new areas of the search space, the weights λ_c used for the penalty terms in the objective function, are reinitialized to random values after executing any of the diversification procedures.

5. EXPERIMENTAL RESULTS

The proposed model and solution approach for SF-APP were tested on a realistic data set for a medium size city. The data is provided by AirTel, Montreal. This study

examines a network at two development points. The first point corresponds to the green field scenario and the second corresponds to a network upgrade scenario, where a major increase in network capacity is implemented. In both cases, we compare the configurations deployed by the operator to those produced by the proposed algorithm. The main characteristics of the network are:

- total area: 33.742Km × 35.182Km with a grid resolution of 100m,
- number of relevant test points: 63345,
- test points carrying traffic: green field scenario 13743, network upgrade scenario 18586,
- number of subscribers: green field scenario: 36479, network upgrade scenario: 124942,
- average traffic per subscriber: 0.022Erlang,
- minimum required signal strength: -92dBm.

The candidate antenna set is obtained by compiling a set of antennas that either have been activated or considered for activation by the network engineers. Thus, it is a filtered set of 334 feasible antennas on 107 sites. We have opted not to add more antennas in order to ensure that we only consider feasible cases (increasing the number of antennas would potentially lead to better solutions). Each antenna can be set to a power level from its associated discrete set Δ_i. This set is limited so that $|\Delta_i| = 7$ (including the off state). Even though the equipment has a wider operating set, this limitation would provide a margin for manual fine tuning of the proposed solutions. The path loss matrix for the candidate antenna set is compiled from field measurements (available for only a subset of antennas) and from a radio propagation simulation tool provided by AirTel, Montreal. Traffic data is provided in the form of estimated offered traffic for a subset of antennas. By uniformly distributing the antenna offered traffic among all its covered test points, we obtain test point traffic.

The target performance levels used in defining Ω, are determined so that feasible solutions are at least as good as the operator's solution. For instance, if the given configuration for the green field scenario uses 91 antennas, then the maximum allowed number of antennas for feasible solution in SF-APP is also 91.

All computations were carried out on Pentium III PC, at 500MHz and with 576Mb of RAM. The algorithm was coded in visual C++. For each scenario, we have evaluated the configurations that were actually implemented by the network operator and compared it the optimized solution of SF-APP. These solutions were obtained after running the tabu search algorithm for three hours in the green field scenario and five hours for the network upgrade scenario.

The results for the two scenarios are summarized in Table 2. Under each scenario, the first column refers to the operator's APP solution and the second column refers to SF-APP. For the network upgrade scenario, we investigate the effect of changing the maximum number of TRX from 4 to 7: for SF-APP, under the network upgrade scenario, the first solution corresponds is generated with a limit of 4 TRXs per antenna and the target performance is set to match the operator's solution (except for network coverage). The second solution, the only requirement is to carry as much traffic as the operator's solution with a limit of four TRXs per antenna, while for the third solution, the limit on the number of TRXs per antenna is increased to seven. Table 2 reports the performance of the solutions for the system criteria presented in Section 3.

Table 2: Optimization results

	greenfield scenario		network upgrade scenario				
APP solution	Operator	SF-APP	Operator		SF-APP		
Max Nb TRXs per antenna	4	4	4	7	4	4	7
% Coverage	45.3	50.4	96.5	96.5	82.9	91.5	86.5
% Traffic coverage	81.9	84.4	99.5	99.5	98.1	98.7	99.1
# Antennas	91	47	183	183	149	152	125
# Sites	34	30	66	66	65	72	67
# TRX	194	147	485	572	454	456	502
% Excess capacity	30.4	16.2	16.0	17.8	11.5	13.1	11.7
% Blocked traffic	3.5	2.9	23.4	6.1	20.9	22.70	6.1
% Carried traffic	79.1	81.9	76.3	93.4	77.6	76.3	93.0
Average CIR	16.0	110.7	155.9	155.9	203.4	159.9	271.8
Average weighted CIR	67.3	195.1	43.4	43.4	211.7	190.9	241.9
% Traffic CIR<12dB	63.0	52.8	82.8	82.8	80.3	73.7	71.8
% Area CIR<12dB	35.3	32.6	75.3	75.3	61.1	65.7	62.2

In the first scenario SF-APP outperforms the operator's solution by reducing the number of equipment used, improving resource utilization, and reducing interference levels. In the network upgrade scenario, examining the operator's solution, we see the effect of the limit of 4 TRXs per antenna. Increasing this limit, results in dramatically increasing the carried traffic and the number of TRXs required to provide that service. Under the original conditions specified by the operator, SF-APP (solution 1) results in a solution with a reduced interference levels, fewer resources and better resource efficiency.

When we relax the constraints on network performance and maintain only the requirement on carried traffic (solution 2), SF-APP further improves the interference level but with slightly more resources. Interestingly, this second solutions has a lower percentage of traffic with low CIR than the first solution but deteriorates the average CIR. The third solution corresponds to increasing the limit on the number of TRXs per antenna. This solution displays further improvements in interference measures with fewer resources than the operator's solution. This solution also appears to provide the best performance in terms of the increased traffic carried and resource utilization. These gains have to be weighed against the increase in costs associated with the installation of combiners (equipment that allows an antenna to handle seven instead of four TRXs). These experiments highlight the flexibility that the model offers. SF-APP allows the user to perform a wide range of sensitivity analysis experiments and to evaluate the tradeoffs involved.

6. CONCLUSION

We have proposed a flexible model for the antenna positioning problem. The model integrates a variety of design criteria that are of interest to network operators. Applied to two problem instances, the proposed heuristic algorithm was able to improve upon the solutions generated by the operator, highlighting the appropriateness of optimization to managing this complex problem for realistic networks. In addition, the results show that the model and algorithm flexibility can provide a mechanism for performing sensitivity analysis.

Future work involves the introduction of a more precise cost function that differentiates between the cost of new equipment and the cost of relocating an existing equipment and providing an accurate characterization of the handoff process and incorporating such a feature within the APP. Another avenue of research which has not be explored much in the literature is the integration of the APP with the other network design components such as designing the fixed component of the network or frequency allocation. These very complex problems are critical to the network performance and are heavily related to the APP.

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